Math 129

Disclaimer:
It is not a good idea to rely exclusively on reading through old exam solutions as a way to prepare for the final exam. Because topics differ slightly from semester to semester and final exams are written with a different flavor from semester to semester, you should not use an old exam to gauge the content of this semester's final exam. This exam can be used as additional practice AFTER completing the department final exam study guide.

1. (8 pts) Calculate the integral. $\int \frac{y^2+5}{y^2+9}\,dy$

2. (5 pts) The rate at which the number of people being infected by a particular virus is proportional to the number of people who have not been infected by time t. There are 500 people in the population and 20 people are initially infected.

Which of the following represents the differential equation for p, the number of people infected by time t? Assume $k > 0$. Mark your answer with an X.

_____ $\frac{dp}{dt} = k(p-500)$ _____ $\frac{dp}{dt} = k \cdot p$ _____ $\frac{dp}{dt} = k(500-p)$

_____ $\frac{dp}{dt} = k \cdot p + 20$ _____ $\frac{dp}{dt} = k(p-500)+20$ _____ $\frac{dp}{dt} = k(p+20)$

3. (8 pts) Calculate the integral. $\int \arcsin(3x)\,dx$

4. (5 pts) Which of the following integrals is equivalent to $\int x^2\sqrt{4+x^2}\,dx$ if you make the trigonometric substitution $x = 2\tan\theta$? Mark your answer with an X.

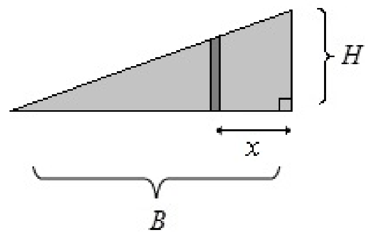
_____ $8\int \tan^2\theta\sec\theta\,d\theta$ _____ $16\int \tan^2\theta\sec^3\theta\,d\theta$ _____ $16\int \tan^3\theta\sec^2\theta\,d\theta$

_____ $8\int \tan^2\theta\sec^2\theta\,d\theta$ _____ $16\int \tan^2\theta\sec^2\theta\,d\theta$ _____ $8\int \tan^2\theta\sec^3\theta\,d\theta$

5. (10 pts) Use the method of partial fractions to calculate the integral. Remember to show all work.

$$\int \frac{t^2+3}{t\left(t^2+1\right)}\,dt$$

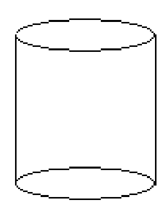
6. (12 pts) Consider the triangular region with dimensions B and H as shown below.



a) Write a Riemann sum approximating the area of the triangular region using vertical strips as shown. Your answer will also have B and H in it.

b) Suppose x, B and H are measured in meters and the density of the triangle in kg/m^2 is proportional to x. Set up, but do not evaluate, an integral representing the total mass of the triangle. Include units in your answer.

7. (10 pts) A cylindrical tank with height 12 feet and radius 2 feet is 3/4 full of water. Set up, but do not evaluate, the integral representing the amount of work needed to pump all of the water to a point that is 5 feet above the top of the tank. The density of water is 62.4 lbs/ft^3 . Include a sketch of your slice and label the variable for your set up.



8. (15 pts) Suppose f and g are continuous functions. If $0 < f(x) < \frac{1}{x^3}$ and $\int_0^\infty g(x)dx$ converges, determine if the following improper integrals converge or diverge. Mark your answer with an X.

a) $\int_1^\infty f(x)dx$ _____ converges _____ diverges _____ impossible to tell

b) $\int_5^\infty g(x)dx$ _____ converges _____ diverges _____ impossible to tell

c) $\int_0^\infty \left(3+g(x)\right)dx$ _____ converges _____ diverges _____ impossible to tell

9. (10 pts) Consider the integral $\int_0^2 \frac{1}{(x-1)^{4/3}}dx$.

a) Explain why the integral is improper.

b) Determine if the integral converges or diverges. You must justify your answer.

10. (10 pts) Suppose the power series $\sum_{n=0}^{\infty} C_n(x-3)^n$ converges for $x=-1$ and diverges for $x=10$. Determine if the power series converges or diverges for the following values. Mark your answers with an X.

a) $x=3$ _____ converges _____ diverges _____ impossible to tell

b) $x=8$ _____ converges _____ diverges _____ impossible to tell

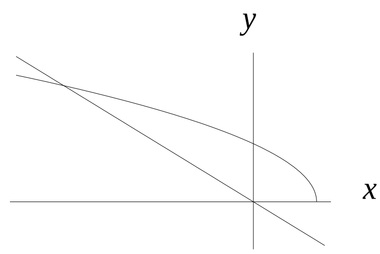
c) $x=-7$ _____ converges _____ diverges _____ impossible to tell

11. (10 pts) a) Use the Taylor series about 0 for $\ln(1+x)$ to find the Taylor series about 0 for $\ln(1-x^2)$. Write out at least 3 terms and simplify.

b) Find the first three non-zero terms of the Taylor series for $\int_0^x \ln(1-t^2)dt$ for $0<x<1$.

12. (12 pts) Consider the region bounded by $f(x) = e^{-kx}$ and the x-axis for $x \geq 0$. Assume k is a positive constant. Revolve this region around the x-axis. Find the exact volume of the resulting solid.

13. (10 pts) Consider the region bounded by $y = -x$, $y = \sqrt{16 - 6x}$, and $y = 0$.

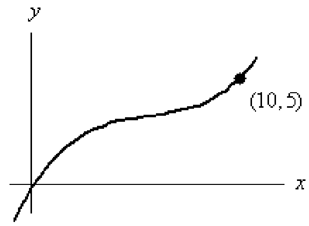


Set up, but do not evaluate, an integral representing the <u>area</u> of the region using strips perpendicular to the *y*-axis. Include a sketch of a slice on the figure above.

14. (10 pts) Find the value(s) of B for which $y = \sin(Bt)$ is a solution to the differential equation

$$\frac{d^2y}{dt^2} + 7y = 0.$$

15. (10 pts) The graph of $y = f(x)$ is shown below.



a) On the graph above, shade the enclosed region bounded by $y = f(x)$, $y = 5$, and the *y*-axis.

b) Rotate the region in part a) around the line $y = 5$. Set up, but do not evaluate, the integral representing the volume of the solid. You do not need to find a formula for $f(x)$.

16. (10 pts) Determine if the series $\sum_{n=1}^{\infty} \frac{(-1)^n 3^{n+1}}{(n-1)!}$ converges or diverges. Remember to show all work.

17. (15 pts) Determine if each of the following converges or diverges. You must justify your answer.

a) The **sequence** $a_n = \dfrac{3n^3 - 4n}{2 + 5n^3}$.

b) The **series** $\displaystyle\sum_{k=2}^{\infty} \frac{2^k}{5^k}$

c) The **series** $1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \dfrac{x^4}{4!} + \cdots$

18. (15 pts) Solve the differential equation subject to the initial condition. Express your solution in explicit form.

$$\frac{dz}{dy} = \frac{yz}{\ln z} \qquad z(0) = e$$

19. (10 pts) a) Use the information below to find the second degree Taylor polynomial that can be used to approximate f near $x = 1$. $f(1) = 3$, $f'(1) = -2$, and $f''(1) = 5$

b) Approximate $f(1.1)$ using your answer in part a).

20. (5 pts) The table below gives a few values for $f(x)$ and $f'(x)$.

x	-1	0	1	$\pi/2$
$f(x)$	10	8	7	1
$f'(x)$	4	3	9	7

Evaluate the integral. Show all work clearly.

$$\int_0^{\pi/2} \sin(2x) \cdot f'(\cos(2x))dx .$$